Performance Based Learning and Assessment Task

Staircases and Ramps

I. **ASSESSSMENT TASK OVERVIEW & PURPOSE:**
The students will use slope to describe the characteristics of ramps versus the stairs between the same two levels of elevation. After practical observation of stairs and ramps, the students will discuss the physical reasons for stairs to be steeper than ramps. Teachers will direct this discussion into talking about the slopes of the stairs and ramps. Students should measure these stairs and the corresponding ramp and transfer their information onto a graphical representation of each. Students will then analyze the slopes and equations of the lines represented by the stairs and the ramps and compare them. After an analysis, students should be able to write an explanation of the relationship of the slope of the lines to the heights and lengths of stairs and ramps and be able to answer teacher questions in an interview.

II. **UNIT AUTHOR:**
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III. **COURSE:**
Algebra I

IV. **CONTENT STRAND:**
Algebra

V. **OBJECTIVES:**
- The learner will be able to determine why ramps are longer than stairs.
- The learner will be able to calculate the slope of both the stairs and the ramp.
- The learner will be able to illustrate graphically how slope effects the length of ramps and stairs.

VI. **REFERENCE/RESOURCE MATERIALS:**
Stairs and corresponding ramps, Graph paper, Graphing Calculator, and Measuring tapes

VII. **PRIMARY ASSESSMENT STRATEGIES:**
The task includes an assessment component that performs two functions: (1) for the student it will be a checklist and provide a self-assessment and (2) for the teacher it will be used as a rubric. Teachers will look at two aspects of the students' work and comprehension. First the teacher will look at the graphs and written explanations from the students and follow the assessment list for this portion. Second the students will be called by group to participate in a small group interview by the teacher in which the teacher will have the students verbally describe to them what is happening between the stairs and ramps. The teacher will use the questions attached as guidelines and the assessment list for the interview.

VIII. **EVALUATION CRITERIA:**
Assessment lists for the paper and interview portions, and corresponding rubrics

IX. **INSTRUCTIONAL TIME:**
90 – 180 minutes.

Strand
Algebra

Mathematical Objective(s)
- The students will make connections between graphical and real-world representations.
- The students will work with slope to understand the aspects of it.
- The students will investigate the nature of slope and how it affects linear equations.

Related SOL

- SOL A.4: The student will solve multistep linear and quadratic equations in two variables, including
  - d) solving multistep linear equations algebraically and graphically;
  - f) solving real-world problems involving equations and systems of equations.
  Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

- SOL A.6: The student will graph linear equations and linear inequalities in two variables, including
  - a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and
  - b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

NCTM Standards

- Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations;
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases;
- Use symbolic algebra to represent and explain mathematical relationships;
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- Draw reasonable conclusions about a situation being modeled.
- Approximate and interpret rates of change from graphical and numerical data.

Materials/Resources
- Staircase with corresponding ramp
- Measuring tape
- Graph paper
- Graphing calculator
- GeoGebra or Excel (optional)

Assumption of Prior Knowledge
- Students should have an understanding of the differences between stairs and ramps.
- The students should be familiar with the concept of slope.
- The students should be familiar with the rate of change.
- The students should know the slope intercept form of linear equations.
- The students should be able to graph points and linear relationships.

Introduction: Setting Up the Mathematical Task

In this activity, your students will investigate the relationship between staircases and ramps and how the corresponding slopes affect the characteristics of each. As a class, you will want to look at a set of stairs and its corresponding ramp. Lead a discussion with the students about the differences between stairs and ramps. The less you tell them allows them to discover and answer the questions for themselves. Some things will remain to be speculation. One illustration you could make if possible would be to place a piece of plywood or a plank over the stair and have the students experiment with an empty wheelchair or a wheelchair with books in it to give weight. This is a good illustration to help the students understand that it is necessary for ramps to be less steep. Some questions you could ask to start discussion are as follows: Have you ever noticed that ramps take up a lot more space than a corresponding set of stairs? Why is that? We will discuss why ramps are longer than stairs in real-world situations and terminology.

After the main discussion, divide the students into groups to make measurements on a set of stairs with a corresponding ramp. They will transfer these measurements into a graphical representation of both the stairs and the ramp. The length is represented by the horizontal change covered by the stairs or ramp. Likewise, the height is the vertical change covered by the stairs or ramp. If you have access to more than one set of stairs with ramp, it might be a good idea to have each group measure different ones. It is also a good idea to have each group measure multiple stairs and ramps for comparison between the groups. Some schools do not have access to this, so focus on the slope if you don't have a multiple set options. If you do have this option, you can always extend the activity to see if the students can figure out the construction regulations for ramps. The students are always excited if they can use their math skills to figure out real world things like that. The students will have the worksheets that are attached to help guide them. There is also an Extended Worksheet to guide the students to estimating the ADA regulations for ramps. The ADA regulations for ramps is a slope of no more than 1/20.

Once they have completed the worksheet and graphs, have each student write two full paragraphs explaining what they discovered and their thoughts on it. You should take each group aside as they finish and perform an interview with them to see if they truly understand what they discovered. A set of suggested interview questions is attached. The purpose of this activity is to have the students come to a greater understanding of slope and how it affects linear relationships using real world examples. If understood properly as a ratio, students should be able to determine the length of any set of stairs or ramp given a height needed to connect. After the activity, teachers should help the students transition this information to the normal slope and linear equation problems they see in the textbook.

Student Exploration
Small Group Work

The students will gather in their groups to measure important characteristics of both the ramp and stairs, preferably of a couple of sets of stairs and ramps. Using the data that they gather, they should make pictorial representations of each. The students should then analyze their graphs to make slope and linear equation

connections for the stairs and the ramps. They should discuss among themselves the similarities and differences they notice between different sets of stairs and ramps, and what role slope plays in their descriptions. The group should then write a two paragraph essay describing their observations and conclusions. Finally, the students should be prepared to discuss how the slope affects the characteristics of the ramp and to make predictions on the length of a ramp when given the height of a set of stairs. These topics will be discussed in an interview with the teacher.

Whole Class Sharing/Discussion
The whole class will begin this lesson with a discussion about real-world situations surrounding stairs and ramps. Why do ramps exist? Why are ramps longer than stairs? Why are they not as steep? If someone were to smooth out a staircase to make a ramp, what would be the repercussions for those using the ramp? As explained in the Introduction, a mention of standard construction regulations can be mentioned in the beginning or as an extra piece of information that the students may discover throughout the activity.

After the interviews have been completed, have the whole class revisit the earlier discussion, but this time, have them discuss the same questions about stairs and ramps using the mathematical terminology to illustrate their connections. If you were able to have the students work with multiple sets of stairs and ramps, have the students share their results at this time and come to a consensus concerning the possible regulations for ramps. Either have them research the ADA regulations for ramps or pull it up on the board as a teacher. Students should discuss if their consensus was correct when compared with the ADA regulations. If not, have the students determine which ramps did not meet the guidelines set forth by the ADA.

Student/Teacher Actions:
- Students should be gathering data and analyzing graphical representations of stairs and ramps for mathematical connections.
- Students should make pictorial and graphical representations of the stairs and ramps and then analyze them to find slope and linear equations.
- Students should discuss in groups the effects of slope on the length of the ramp, and write one or two paragraphs explaining their observations.
- Teachers should be moving between the groups to facilitate learning within the groups. They should be available to answer questions and use their own questioning to prompt students to higher levels of thinking.
- Teachers should also take groups that are finished with the analysis aside for group interviews to assess learning.

Monitoring Student Responses
Students will be writing an explanation of their findings and reasoning and submitting this to the teacher. Upon completion of the calculations and explanation, the groups will be taken out for an interview with the teacher to see if they can verbally explain and represent their findings. Suggested questions for the teacher interview portion are attached.

Assessment List and Benchmarks
During the discussion and measurements portion of the assignment, the students will compile at least two graphical representations of a staircase and a ramp (possibly more of each). These will be taken up by the teacher and graded for clarity and accuracy. The students will then complete the written explanation portion which will be graded according to the attached rubric. Likewise, the interview portion will be graded according to the attached rubric. There is no rubric for the Extended Worksheet, as many teachers may not be able to use it. This is intended as discussion and can be graded to the teacher's discretion.

Student Worksheet

In this activity, you will investigate the relationship between staircases and ramps and how the corresponding slopes affect the characteristics of each. As a class, we will look at a set of stairs and its corresponding ramp. Have you ever noticed that ramps take up a lot more space than a corresponding set of stairs? Why is that? We will discuss why ramps are longer than stairs in real-world situations and terminology.

As a group, you will measure several important aspects of both the stairs and the ramp. The height is measured by the distance between the two levels traversed by the stairs and ramp. The length is the horizontal distance between the starting point and ending point of each, not the slant length. You will take these measurements to your graph paper to make pictorial representations of the stairs and ramps. Using slope and linear equations, list as many aspects of the stairs and ramps as you can and see how they relate to each other. You will then write two paragraphs about your findings, and use your information and new understanding to find ramp lengths that correspond to stair heights. As a final assessment, your teacher will interview each group to assess full understanding of the relationships between stairs and ramps with regard to slope.

Use each of the following questions to guide you in your investigation:
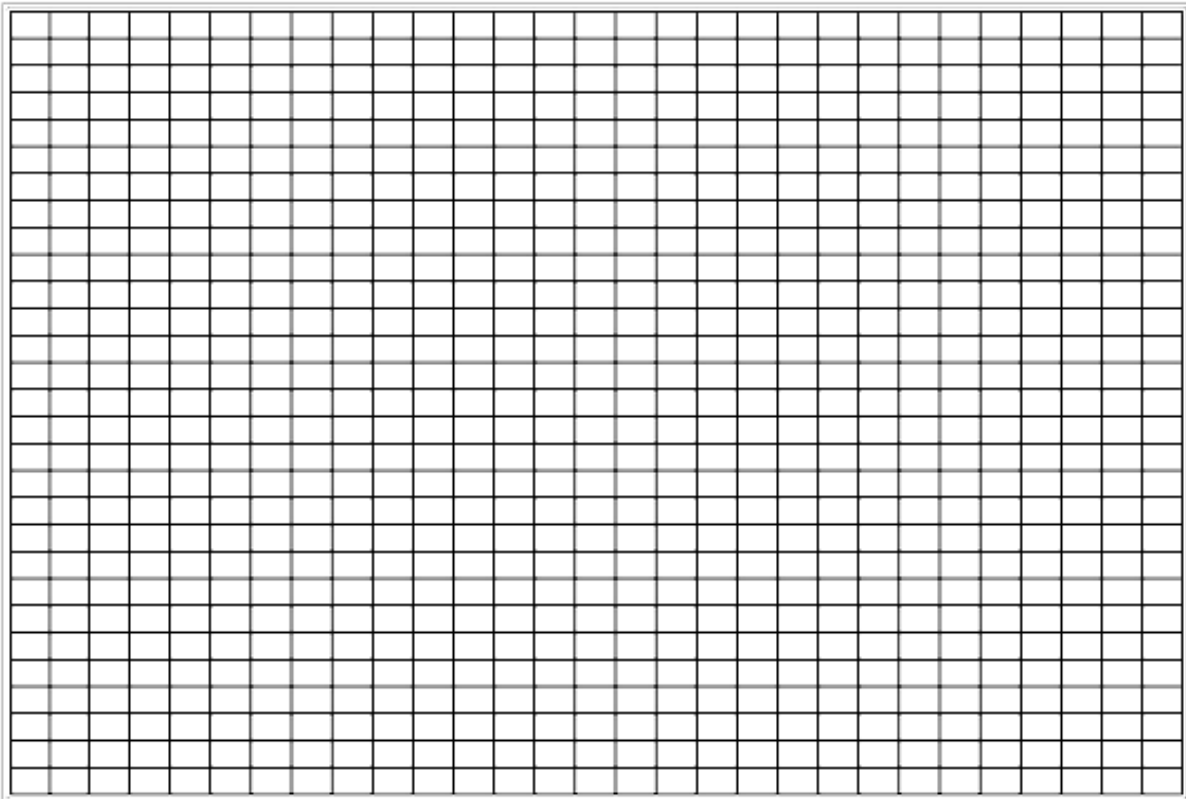
- What is the height of stairs? How did you determine this?

- What is the length of stairs? How did you determine this?

- What is the height of ramp? How did you determine this?

- What is the length of ramp? How did you determine this?

- What is the slope of each?

- What is the equation of each line?

Student Graphical Representations

Stairs
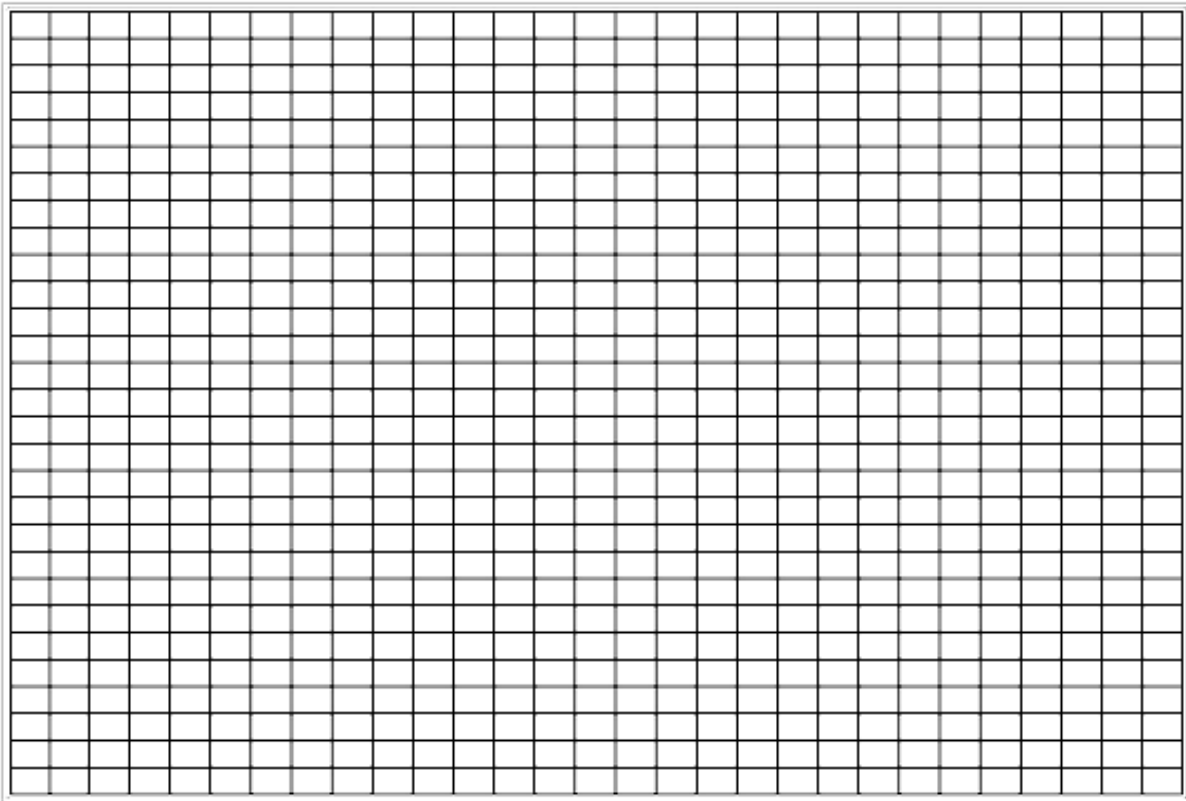
Ramp

Stairs and Ramps Written Portion Rubric

The following rubric and category descriptions will be used to assess your paragraph explanations. Please review the rubric carefully.

Number	Element	Point Value	Self-Graded	Teacher Grade
1	The calculations are complete.	5		
2	The writing is the appropriate length.	3		
3	The explanation is well organized and neat.	3		
4	The explanation is coherent and well written.	4		
5	Details are given to back up the students' explanations.	6		
6	The explanation was well thought out.	5		
7	Connections were made to help with understanding slope in a deeper context.	4		
	Total	**30**		

Written portion

	Element				
1	The calculations are complete.	**0** No calculations were completed.	**1** Less than half of the calculations were completed.	**3** At least half of the calculations were completed.	**5** All of the calculations were completed.
2	The writing is the appropriate length. (Two paragraphs of at least three sentences each.)	**0** No writing was done.	**1** Less than half of the required length.	**2** At least half of what was required.	**3** The writing was the appropriate length.
3	The explanation is well organized and neat.	**0** No explanation given.	**1** Explanation given but barely legible.	**2** Explanation is legible but unorganized.	**3** Explanation is neat and organized.
4	The explanation is coherent and well written.	**0** No explanation given.	**2** Explanation hard to follow.	**3** Explanation is good but a little confusing.	**4** Explanation is clear and good.
5	Students use specific examples of their work to back up their explanations.	**0** No examples were given.	**2** Only one example given.	**4** A few examples were given.	**6** Examples were given for each part of the explanation.
6	The explanation was well thought out.	**0** No explanation was given.	**1** Explanation is too simple.	**3** Explanation is good but more thought could be there.	**5** Explanation was very well thought out.
7	Connections were made to help with understanding slope in a deeper context. (as a ratio, between slope and linear relationships, etc.)	**0** No connections were made.	**1** Connections didn't apply to the lesson.	**2** A connection was made weakly.	**4** Connections were strong.

Stairs and Ramps Interview Portion Rubric

The following rubric and category descriptions will be used to assess your paragraph explanations. Please review the rubric carefully.

Number	Element	Point Value	Self-Graded	Teacher Grade
1	Students were polite during the interview.	3		
2	All group members participated in the interview.	3		
3	All of the questions were answered.	3		
4	The answers given were well thought out.	6		
5	Students made connections between slope and the characteristics of stairs and ramps.	6		
6	Understanding of the lesson was achieved.	3		
7	The students felt confident in their answers and conclusions by the end of the interview.	6		
	Total	**30**		

Written portion

	Element				
1	Students were polite during the interview.	**0** Students were rude and uncooperative.	**1** Only one student would cooperate.	**2** Over half of the students would not cooperate.	**3** All of the group were polite and cooperative.
2	All group members participated in the interview.	**0** No one participated.	**1** Only one student would speak.	**2** Over half of the students would speak.	**3** All of the group participated.
3	All of the questions were answered.	**0** No questions answered.	**1** Only a couple of questions were answered.	**2** Over half of the questions were answered.	**3** All of the questions were answered.
4	The answers given were well thought out.	**0** No thought was given.	**2** Some answers were thought about.	**4** Most of the answers were thought about.	**6** All answers were well thought out.
5	Students made connections between slope and the characteristics of stairs and ramps.	**0** No connections were made.	**2** Connections didn't apply to the lesson.	**4** A connection was made with slopes of stairs and ramps, but nothing further.	**6** Connections were made with slopes of stairs and ramps, and slope in more general situations.
6	Understanding of the lesson was achieved.	**0** No understanding was achieved.	**1** Some understanding was achieved.	**2** Most of the understanding was achieved.	**3** Understanding of the lesson was achieved.
7	The students felt confident in their answers and conclusions by the end of the interview.	**0** No confidence was displayed.	**2** Some confidence was displayed.	**4** Students were mostly confident.	**6** Students were very confident.

Suggested Teacher Interview Questions

1) How did you transfer your data into a pictorial or graphical representation?

2) How did you calculate the slope and linear equation for each graph?

3) What did you notice about the slope and the height versus length of the stairs and ramps?

4) What will happen to the ramp if the height of the stairs changes? For example, what if the new height is 6 ft?

5) Using the same scenario as #4, does the length of the stairs change?

6) Why is the slope less for a ramp versus a set of stairs?

7) What can you tell me about slope based on your observations during this activity?

8) What other observations did you make during this activity?

Extension Worksheet

The Americans with Disabilities Act (ADA) has set forth guidelines and regulations regarding the slopes of ramps. Share the slopes that you calculated for your ramp(s) with the class and gather all of their slopes. Discuss in your groups and as a class the calculated slopes and answer the following questions.

1) What were the calculated slopes for the class? List them here.

2) Do you notice any patterns or similarities between the slopes of the different ramps?

3) Are there any consistencies? Inconsistencies?

4) Based on your observations, what do you think the ADA regulations are?

5) After researching, what are the ADA ramp regulations?

6) Were your guesses correct? Why or why not?

7) Did all of your ramps meet the ADA regulations?

Sample Work: (Actual numbers may vary.)

In this activity, you will investigate the relationship between staircases and ramps and how the corresponding slopes affect the characteristics of each. As a class, we will look at a set of stairs and its corresponding ramp. Have you ever noticed that ramps take up a lot more space than a corresponding set of stairs? Why is that? We will discuss why ramps are longer than stairs in real-world situations and terminology.

As a group, you will measure several important aspects of both the stairs and the ramp. The height is measured by the distance between the two levels traversed by the stairs and ramp. The length is the horizontal distance between the starting point and ending point of each, not the slant length. You will take these measurements to your graph paper to make pictorial representations of the stairs and ramps. Using slope and linear equations, list as many aspects of the stairs and ramps as you can and see how they relate to each other. You will then write two paragraphs about your findings, and use your information and new understanding to find ramp lengths that correspond to stair heights. As a final assessment, your teacher will interview each group to assess full understanding of the relationships between stairs and ramps with regard to slope.

Use each of the following questions to guide you in your investigation:

- What is the height of stairs? How did you determine this?

  31 inches. We measured from the ground to the top level of the stairs, since it was a short set of stairs.
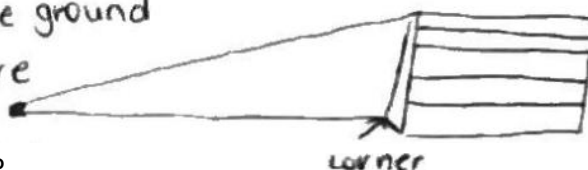
- What is the length of stairs? How did you determine this?

  61 inches. We measured from the corner (shown in the picture) to the front of the set of stairs. This gave us the length which was the change in x for the stairs.

- What is the height of ramp? How did you determine this?

  31 inches. Since the ramp and the stairs are built to reach the same level, the heights will be the same.

- What is the length of ramp? How did you determine this?

  303 inches. We measured the distance from the point where the ramp met the sidewalk to the corner keeping the tape measure next to the curb. This gave us the length.

- What is the slope of each?

  Stairs: $\frac{\Delta y}{\Delta x} = \frac{31}{61} \approx 0.51$. This is very close to a slope of $\frac{1}{2}$.

  Ramp: $\frac{\Delta y}{\Delta x} = \frac{31}{303} \approx 0.10$. This is very close to a slope of $\frac{1}{10}$.
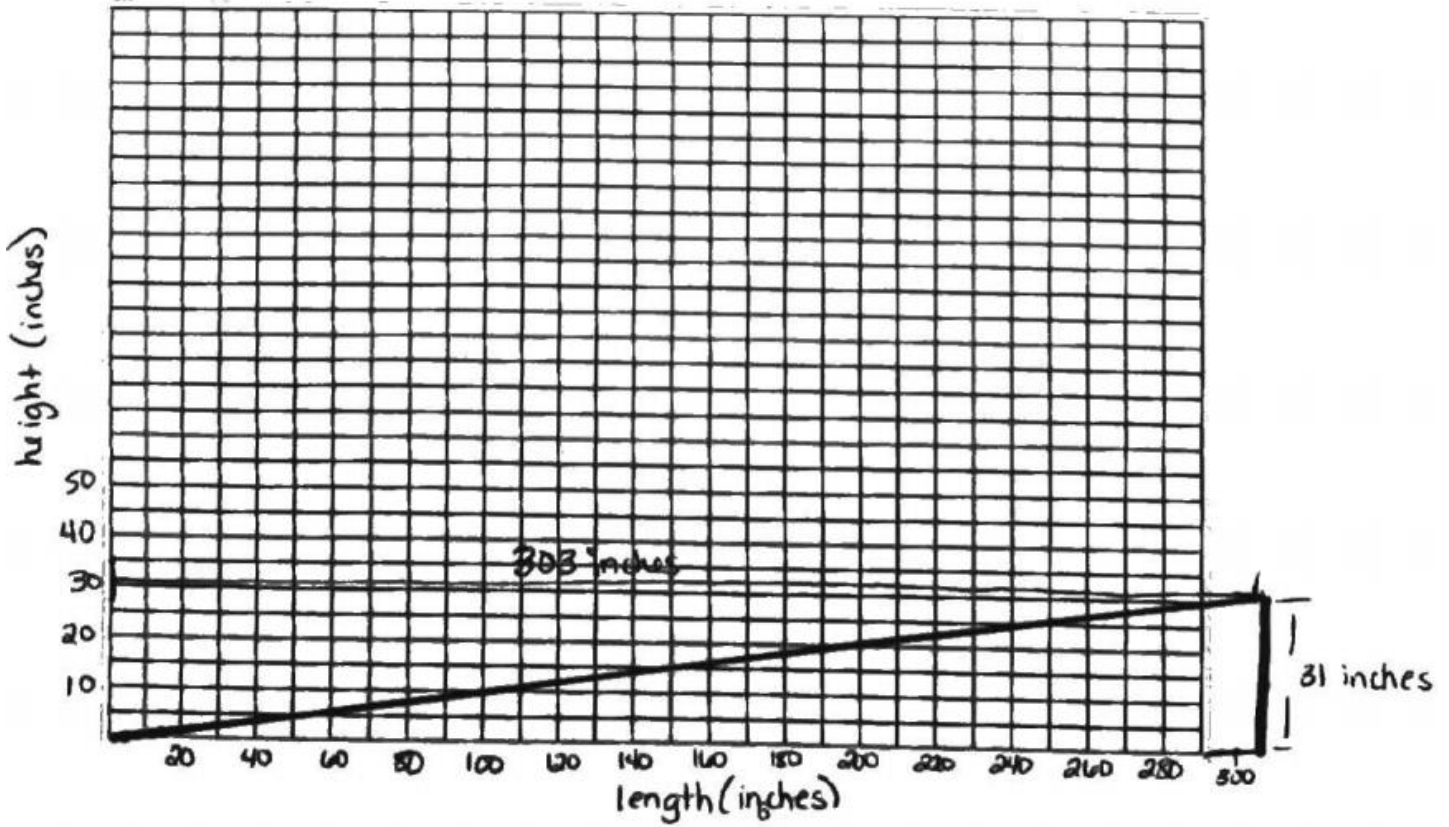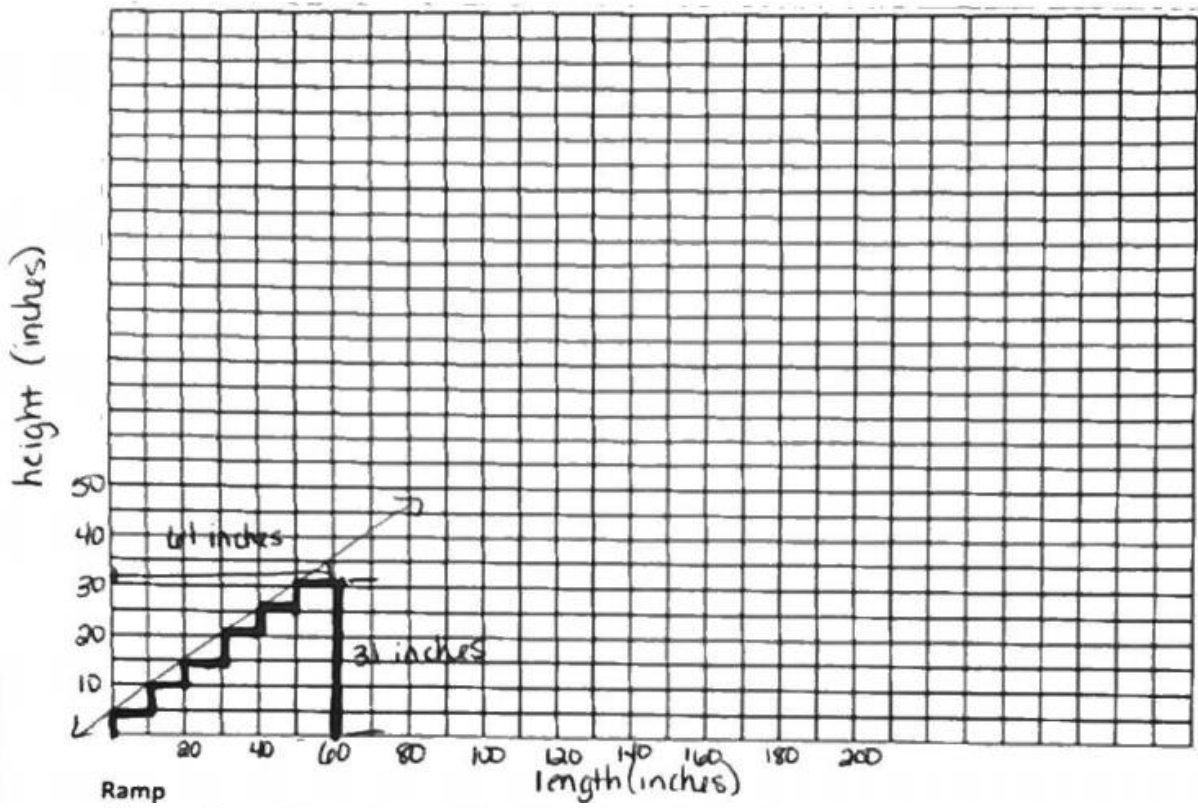
- What is the equation of each line?

  Stairs: $m = 0.51$ and $b = 6.2$ inches. (This is the height of the first step.)
  $$y = 0.51x + 6.2$$

  Ramp: $m = 0.10$ and $b = 0$ inches. (The ramp meets the ground.)
  $$y = 0.10x + 0$$
  or $\quad y = 0.10x$

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Stairs

height (inches)

50
40 61 inches
30
20   31 inches
10

20   40   60   80   100   120   140   160   180   200
length (inches)

Ramp

height (inches)

50
40
30 303 inches
20
10   31 inches

20 40 60 80 100 120 140 160 180 200 220 240 260 280 300
length (inches)

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Suggested Teacher Interview Questions (Desired thought processes are included. Students will be answering in the first person format. Use these guidelines to keep them on the right track.)

1) How did you transfer your data into a pictorial or graphical representation?

   Students should answer this question by discussing how they measured the length and width of the stairs and the ramp. They should have made the connection that the length translated to the x-axis and the height translated to the y-axis.

2) How did you calculate the slope and linear equation for each graph?

   Hopefully, they made the connection that the slant of the ramp and the tops of the stairs made a line that they were then able to calculate the slope and linear equation from. Another connection that could be made would be that the y-intercept for the ramp would be 0 because it starts at the ground. An interesting discussion could be had about the y-intercept for the stairs. Some would argue that the stairs start at the ground as well, while others will say that since the line was made by connecting the top of the stairs, the y-intercept should be the top of the first step.

3) What did you notice about the slope and the height versus length of the stairs and ramps?

   Here the students should make the connection that since the slope of the stairs is greater than that of the ramp, for any given height the stairs will have a shorter length than the ramp. Further, observations may also be made as to the relationships between the lengths and heights.

4) What will happen to the ramp if the height of the stairs changes? For example, what if the new height is 6 ft?

   What we are looking for here is a connection that the height of the stairs and corresponding ramp are the same since they are traversing the same two levels. If the height changes, the length of the ramp will change much more than the stairs. The length will be proportional to the height using slope as the ratio. The students will have to use their calculated slope for the ramp to answer the specific example of a height of 6 ft.

5) Using the same scenario as #4, does the length of the stairs change?

   The students should realize here that length will change but not as much as the length of the ramp due to the stairs having a much greater slope. The students will have to use their calculated slope for the stairs to answer the specific example of a height of 6 ft.

6) Why is the slope less for a ramp versus a set of stairs?

   This should be a reiteration of the discussion before the activity. Possible explanations would be that handicapped people wouldn't be able to roll themselves up a ramp that was a steep as the stairs. They need a more gradual incline for safety and practicality. Also, descending a ramp of the same slope as the stairs would be very dangerous and difficult to stop at the end. If the teacher wants a more specific reason, they could have the students add a small research session to answer this question.

7) What can you tell me about slope based on your observations during this activity?

   The connection that is the hope of this entire activity is that slope is a ratio that stays the same no matter where you are on the line. In other words, finding the slope of stairs and a ramp of a set height, the students would then be able to determine the lengths needed for any height needed for the stairs and ramps. Students should be able to take this information back to the textbook problems to further their knowledge of slope and linear equations.

8) What other observations did you make during this activity?

   This is kind of open to teacher discretion on the validity of answers for this problem. This is mainly just a way to let the students mention if they discovered anything outside of slope in the activity. Not only does this question potentially open new avenues of discussion for the activity, the students may make an observation that the teacher never thought of.

   Feel free to add any other questions as these are just suggestions. The answers supplied are also just supplying the line of thinking hoped for in the design of this activity.